How to teach modeling in mathematics classrooms? The implementation of modeling tasks. Comparing learning arrangements and teacher methods with respect to student's activities.

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Abstract

There is a wide consensus that including mathematical modeling into the curricula is an important aim. A lot of attention has been spent on the realistic problems whereas their embedding in a classroom situation is less investigated so far, although the methodical arrangements are of major importance for initiating students' activities. In this paper, the implementation of the modeling task "swimming pool" in mathematics education in two lessons is compared concerning learning arrangement and teaching methods in depth with respect to the students' activities. Two videos about this implementation will be shown and discussed in this workshop. They are supposed to demonstrate in which different ways teachers engage in modeling. The aim is to show teachers, especially those without experience in teaching modeling, how modeling tasks can be implemented.

Introduction

For a certain period of time, there has been an agreement in the pedagogical discussions about the integration of applications and modeling into mathematics curricula (Kaiser-Messmer, 1986(I)) in order to enable students to understand and think critically about their environment, as well as about the usefulness of mathematics in society (Blum & Niss, 1991). Modeling is not only one of the five competences, which are important to teach in German schools. It enables practicing the other competences as well (Maaß, 2006). During modeling the students have to solve problems, discuss, communicate and illustrate their solutions. They can realize and use the relating mathematics in order to cope with the modeling tasks that come from their environment. Hence, the modeling competence is necessary for the students to be able to manage their daily life and to prepare for their later working life.

Project intention

This article is embedded in the project "Developing Quality in Mathematics Education II", called DQME II. A central aim of this project is to link theory and practice. 34 partner institutions like universities, institutions of teacher training and schools are working within this project. The participants (teachers, teacher trainers, researchers) develop material, especially concerning mathematical modeling for teachers in Europe. The developed tasks are evaluated in their own country, edited and internationally spread between the cooperating schools afterwards. Cross-cultural cooperation and exchange of ideas, materials and methods between eleven European countries ensure a successful development of intercultural education.

Methodology

Video-sequences enable to get a better understanding of the development of epistemological aspects of mathematics education. The evaluation of the tasks within this project is videotaped. The video and the modeling task I present in this paper emerged from this project. The original task has been developed and tested in Sweden. From there it has been sent to a participating school in Germany. One of our German teachers modified this task:

Swimming-Pool

Despite the rather cool weather during winter, small outdoor swimming pools are popular among private house owners in Germany. Imagine a swimming pool that is circular with a radius of 2.75 meters and a depth of 1.18 meters. The distance between the water surface and the pool edge is 0.06 meters. Every spring the pool is filled through two water pipes, each of them delivering 20 liters of water per minute. The water cost 2 Euro per cubic meter.

1. Figure out at least 2 meaningful mathematical questions and answer them with a calculation.
2. Find an agreement in your group. Which one of your questions should be written on the blackboard?
3. Hence, solve the questions of the other groups.

Figure 1: The Swimming-pool task

The task in figure 1 was given to five participating teachers in Germany. The requirements were identical. The teachers got the same short instruction: “Please give a lesson with this worksheet!” The implementation, how to teach it, was decision of the teachers. They were allowed to use the teaching methods they preferred. The teachers also did not know how other teachers designed their lessons around the task.

The empirical data emerged from classroom observations in different grades (7-10) in Germany. Every class had between 28 and 30 students, so that we can speak about an authentic classroom situation. The classroom communication was videotaped. The videographers were observers; they were told to take no influence on the classroom communication and on the teacher’s way to introduce the concepts. Altogether, five classes were visited and two will be compared here. The qualitative interpretation of data is founded on an ethnomethodological and interactional point of view (cf. Voigt, 1984; Meyer, 2007).

Descriptors

Teachers have to learn how to teach modeling because of the embedding of modeling in mathematics curricula and the necessity of modeling competence. Therefore teachers need to know which abilities students need for modeling. The “modeling descriptors” are developed in DQME II and describe students’ abilities (see table 1).

Observation

Two different teachers used the given task in two different ways including different teaching methods. For a better understanding I will use the synonyms teacher A and teacher B. Teacher A gave the students the worksheet and told them that this is their task for this lesson.

In contrast to that, teacher B explained where the task came from, then he made all students read it and afterwards the assignment of the task was summarized by one student.

Learning Objectives		descriptors
Systematisation & Mathematisation	a	Is data needed?
	b	Abstraction
	c	Assigning variables
	d	Making assumptions
	e	Simplifying
	f	Representation(s)
Doing the mathematics	a	Formalising and analysing the mathematics problem
	b	Using data
	c	Approximation and estimation
	d	Use of ICT (software and graphics calculator)
	e	Use algorithms
	f	Mathematical common sense
	g	Proof (validation of the mathematics used)
	h	Use of mathematical representations
Interpretation & Validation	a	Validation of the solution mathematically
	b	Validation of the solution in the 'real world'

Table 1: Some descriptors of modeling processes (Results of the first meeting of the research group, 2007)

The teachers helped their students in different ways to find a solution. Teacher B helped the students with information and explained in detail. Teacher A helped his students to validate their solutions. He was waiting for an explanation of the students results. (Systematisation and Mathematisation a)

student: “That cannot be right. On the blackboard, there is written 18 people and we have 869 people getting the pool to overflow.”
teacher: “869 people get the pool to overflow in your answer?”
student: “Yes!”
teacher: “What measure did you take? How did you calculate that?“
student: “To estimate the people, for an adult, I used your height and the width. 60.”
teacher: “Tell me the volume, which you used.“

Here, the student was doing mathematics (doing mathematics g). He validated the mathematical model he used regarding to the real situation (Interpretation and validation a and b). Teacher B answered the question “Which volume does a person have?” with an action. He tied the cable of the overhead projector around himself and helped the students to measure it. Then he explained the students how to proceed. (Systematization and

Mathematisation a) Teacher A only helped when students ask for. The rest of time the students were working in their groups. Teacher B walked around in classroom, offered help and tried to get an overview on the working process.

The teachers A and B also showed similarities: They chose groups of students for the working process. Teacher B divided the students in groups of 4-5 students and teacher A let the students build groups on their own. The groups were of different sizes (2-5 students). Both took the black board for the result validation (Interpretation and validation a and b). The students wrote their questions on the black board and every group wrote their results under the corresponding question.

Figure 2:
Teacher A shows a student solution on the blackboard.

Quite contrary was the validation of the results. Teacher A used the students' solutions on the blackboard and discussed them in the plenum. Teacher B on the other hand showed the overview of the solutions on the blackboard and asked for presentation by each group on the overhead projector. So he invested a lot of time in the presentation. Every solution was analyzed by the whole class. They discussed all processes of finding a solution and analyzed the failures. Meanwhile teacher A asked the whole class for the solution and discussed with the wrong answers in the plenum.

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